Journal Club for Condensed Matter Physics DOI:10.36471/JCCM_January_2025_02
https://www.condmatjclub.org

Expanding Fermi Gas and Black Hole Information

Generalized Hydrodynamic Description of the Page Curve–like Dynamics of a Freely Expanding Fermionic Gas
Authors: Madhumita Saha, Manas Kulkarni, and Abhishek Dhar
Phys. Rev. Lett. 133, 230402 (2024)

Recommended with a Commentary by Masaki Oshikawa ,
Institute for Solid State Physics and Kavli Institute for Physics and Mathematics of Universe, University of Tokyo

Black holes are among the most fascinating objects in physics. In their original conception within classical physics, black holes absorb everything that falls into them, and nothing can escape. However, Hawking showed that black holes actually emit radiation, owing to quantum fluctuations near the event horizon [1]. This result led to the so-called "black hole information paradox": whether the information carried by the matter falling into the black hole is truly lost or not. Naively the information seems to vanish, but that would contradict the basic principle of quantum mechanics: unitarity.

The black hole information paradox shares similarities with the questions about the second law of thermodynamics and thermalization. We all know that most macroscopic processes are irreversible, and their irreversibility is characterized by increasing entropy. Eventually a macroscopic system usually thermalizes, reaching an equilibrium state which maximizes the entropy of the total system. On the other hand, we also believe that the underlying microscopic dynamics is governed by quantum mechanics. Reconciling the irreversibility and thermalization at the macroscopic level with the quantum mechanical unitary time evolution at the microscopic level is an important question in the foundations of statistical physics. For a long time, most scientists avoided studying such a question — it looked too difficult for publishing papers, and the second law of thermodynamics and thermalization just work, once we accept them. Recently, however, these questions are more actively investigated [2, 3], with the modern concepts such as the "eigenstate thermalization hypothesis" (ETH). With the experimental developments in cold atoms and other highly controllable quantum many-body systems, they are no longer just philosophical questions, but often can be addressed by experiments.

Few would argue that the unitarity in quantum evolution fails in macroscopic systems. In contrast, it has been often suggested that the unitarity is violated in the black hole evaporation process. Indeed, this was Hawking's original argument [4]. However, Page took the position that the entire system including the interior of the black hole and the emitted radiation still obeys unitary time evolution in the black hole evaporation process [5, 6]. He analyzed the quantum entanglement between the black hole interior and the radiation, assuming that the entire system is in a pure quantum state. Further assuming that the quantum state is maximally scrambled, it can be represented by a random pure state in the total Hilbert space. Let us denote the dimensions of the total Hilbert space, the black hole interior, and the radiation by 2^N, 2^s, and 2^t, respectively, where $N = s + t$ is a constant. Initially $s = N$ and $t = 0$, meaning there is no radiation. As the black hole emits radiation, t increases while s decreases. The radiation is strongly entangled with the black hole interior; Page showed that the radiation is almost maximally entangled with the black hole interior so that the entanglement entropy $S_E \sim t$ when $t < N/2$. Interpreting the radiation degrees of freedom t as proportional to the elapsed time in the black hole evaporation process, this implies the linear growth of the entanglement entropy of the radiation with time. Eventually, though, the black hole's internal degrees of freedom s become too small to support further growth of the entanglement; S_E begins to decrease after reaching a maximum at $t = N/2$. For $t > N/2$, $S_E \sim s = N - t$ is constrained by the remaining degrees of freedom s in the black hole interior. This dependence of S_E on t is known as the "Page curve", and the time corresponding to $t = N/2$ when S_E peaks is called the "Page time".

In recent years, the more prevailing view on the black hole information paradox is, following Page's argument, that the unitarity is maintained and the original information is preserved in an "encoded" form in the radiation [7]. A key subject of study is how to "decode" the original information from the radiation [8, 9]. It then seems that the black hole information paradox is even more closely related to the problem of the second law of thermodynamics and thermalization than it appeared initially. In any event, the Page curve itself, as originally derived, does not explicitly rely on black hole physics; it rather follows from generic arguments about random pure states in a bipartite system [5]. This raises an intriguing question of whether a phenomenon similar to the Page curve can be observed in other quantum many-body settings.

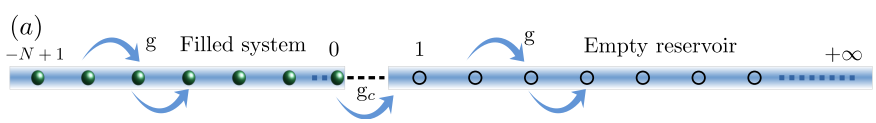

Figure 1: The setup. A half-infinite, one-dimensional tight-binding model with nearest-neighbor hopping g is divided into a finite "system" segment and a half-infinite "reservoir" segment. The two segments are connected by a single junction link g_c. In the initial state, the system segment is filled with fermions, while the reservoir segment is empty. The entire system is then subject to the unitary time evolution under the free-fermion Hamiltonian; the fermions freely expand into the reservoir. (Taken from Fig. 1(a) of the highlighted paper [Phys. Rev. Lett. 133, 230402 (2024)].)

In the highlighted paper, the authors indeed study the entanglement entropy between an "interior" and a "radiation" in a simple free fermion system. They use a one-dimensional tight-binding lattice model where a finite segment (the "system") is connected to a half-infinite segment (the "reservoir") by a single junction link; see Fig. 1. Initially, the finite "system" segment is filled by fermions, while the "reservoir" segment is empty. Allowing the fermions to evolve under the free-fermion Hamiltonian, they expand into the reservoir.

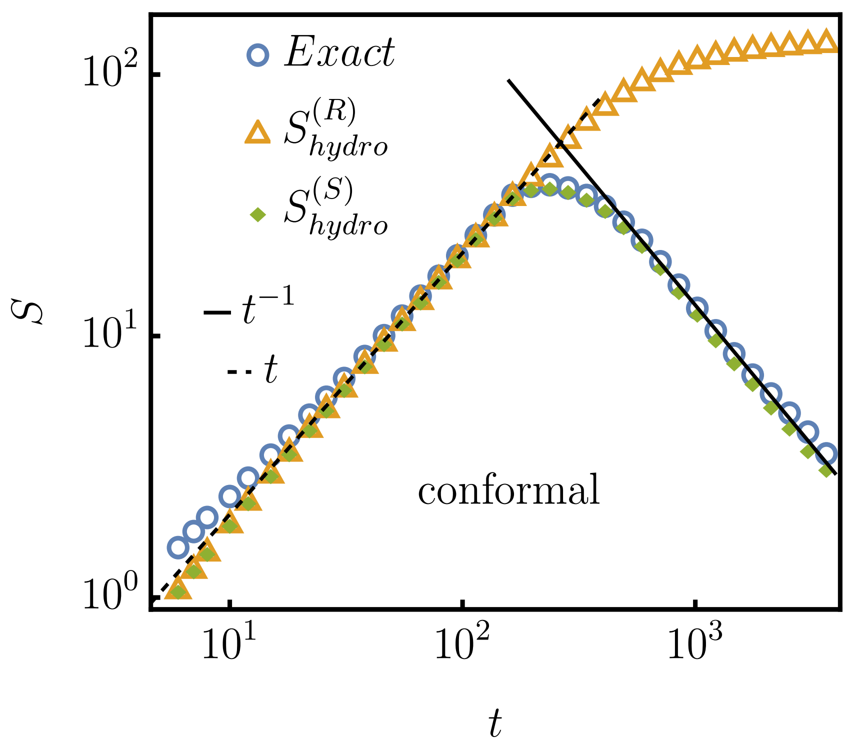

Figure 2: The entanglement entropy and the hydrodynamic entropy of the system and the reservoir, as functions of the elapsed time for a type of the junction link (called "conformal"). The entanglement entropy is obtained exactly based on the free fermion solution of the entire system; it behaves very similarly to the Page curve, with a maximum attained at a finite "Page time". The hydrodynamic entropies of the system and the reservoir also behave similarly to the Page curve, up to the "Page time". After the "Page time", while the hydrodynamic entropy of the system decreases still following the Page curve, the hydrodynamic entropy of the reservoir deviates from the Page curve and continues to increase. (Taken from Fig. 2 of the highlighted paper [Phys. Rev. Lett. 133, 230402 (2024)].)

Taking advantage of the free fermion nature, the authors calculated the entanglement entropy exactly. Its time dependence indeed closely resembles the Page curve; see Fig. 2. Qualitatively, the growth and the decrease of the entanglement entropy is naturally expected; Initially, there is no entanglement between the system and the reservoir because the reservoir is empty. Then the entanglement grows as the fermions spread into the reservoir from the system. Later, as the number of fermions in the system becomes less than that in the reservoir, the entanglement entropy should also decrease proportionally to the number of fermions in the system. Hence, there is a characteristic time ("Page time") when the entanglement entropy is maximal. Nevertheless, the resemblance of the entanglement entropy in the free fermion system to the original Page curve is quite striking.

Entanglement entropy is also relevant to discussion of the second law of thermodynamics and thermalization. While the entanglement entropy should not be confused with the thermodynamic entropy, they could be related. When the system is coupled to an external reservoir and the entire system is in a pure quantum state, the entanglement entropy between the system and the reservoir is nothing but the von Neumann entropy of the reduced density matrix for the system. If the reduced density matrix is close to a thermal equilibrium (Gibbs ensemble), the von Neumann entropy (and thus the entanglement entropy) is equated with the thermodynamic entropy.

On the other hand, in a closed quantum system, the system remains in a pure quantum state following the unitary time evolution, and the von Neumann entropy of the system remains zero. The apparent increase of thermodynamic entropy in the closed system may be understood as arising from coarse graining. If we focus on macroscopic observables, they can be described by a coarse grained density matrix. It does not have to follow the unitary time evolution, allowing its von Neumann entropy to increase.

In the highlighted paper, the authors also use generalized hydrodynamics to study the time evolution for the same setup (Fig. 1). The generalized hydrodynamics describes the state in terms of the phase-space density $n_t(x, k)$ of fermions with momentum k, at position x and time t. The evolution of $n_t(x, k)$ is given by Euler equation. The validity of the hydrodynamic description is demonstrated by the good agreement on the time evolution of the density profiles between the exact results and the hydrodynamic ones.

One can then introduce the hydrodynamic entropy defined in terms of the phase-space density $n_t(x, k)$. This is a coarse grained entropy, as microscopic details of the system are not resolved in the phase-space density. The authors found very intriguing behavior of the hydrodynamic entropy. In the initial stage of the expansion, before the "Page time", the hydrodynamic entropy of both the finite system and the reservoir increases, and follow the "Page curve" of the entanglement entropy. After the "Page time", the hydrodynamic entropy of the system decreases still following the Page curve. In contrast, the hydrodynamic entropy of the reservoir continues to increase (albeit slower than in the initial stage) even after the "Page time".

Hence, while the "radiation" in this free-fermion analog appears to become thermal as its coarse-grained thermodynamic entropy continues to increase (although the free fermion system is ballistic and never fully thermalized in the strict sense), the radiation as a whole consisting of many fermions rather approaches a pure quantum state as all the fermions move into the reservoir eventually. This indeed looks similar to the fate of the black hole radiation which encodes the original information, when the black hole evaporates completely.

These results are very intriguing and motivate further explorations in more generalized or realistic settings. I hope this will stimulate further black-hole inspired studies of quantum many-body systems in statistical mechanics and in condensed matter contexts; hopefully it will turn out to be useful for the black hole physics as well.

I acknowledge with gratitude the discussion meeting "Quantum Many-Body Physics in the Age of Quantum Information" held at International Centre for Theoretical Sciences (ICTS), Tata Institute of Fundamental Research in November 2024, Bangalore, India, for the opportunity to learn about the highlighted work. I also thank Tatsuhiko N. Ikeda, Yoshifumi Nakata, Hal Tasaki, and Beni Yoshida for useful comments on the manuscript.

References

[1] S. W. Hawking, Nature 248, 30 (1974).

[2] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Adv. Phys. 65, 239 (2016).

[3] P. Strasberg and A. Winter, Phys. Rev. X Quantum 2, 030202 (2021).

[4] S. W. Hawking, Phys. Rev. D 14, 2460 (1976).

[5] D. N. Page, Phys. Rev. Lett. 71, 1291 (1993).

[6] D. N. Page, Phys. Rev. Lett. 71, 3743 (1993).

[7] S. Raju, Phys. Rep. 943, 1 (2022).

[8] P. Hayden and J. Preskill, JHEP 09, 120 (2007).

[9] K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, and C. Monroe Nature 567, 61 (2019).