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Observation of dissipationless transport of 1D interacting bosonic atoms

Characterising transport in a quantum gas by measuring Drude weights
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Recommended with a Commentary by Carlo Beenakker, Instituut-Lorentz, Leiden University

The Drude conductivity

$$\sigma_{\text{Drude}}(\omega) = \frac{2iD}{\omega + i/\tau} \tag{1}$$

has a real (hence dissipative) part that tends to a delta function in the limit $\tau \to \infty$ of infinite relaxation time,

$$\lim_{\tau\to\infty} \operatorname{Re}\sigma_{\text{Drude}}(\omega) = 2\pi D\delta(\omega). \tag{2}$$

The weight D of the delta function is referred to as the "Drude weight" (or also as the "charge stiffness"). In the phenomenological Drude model of a gas of free electrons (density n, mass m) one has $D_{\text{free}} = e^2 n/2m$.

More generally, the conductivity of an interacting quantum gas can be decomposed into a delta function contribution plus a term that is regular at $\omega = 0$,

$$\operatorname{Re}\sigma(\omega) = 2\pi D\delta(\omega) + \sigma_{\text{regular}}(\omega). \tag{3}$$

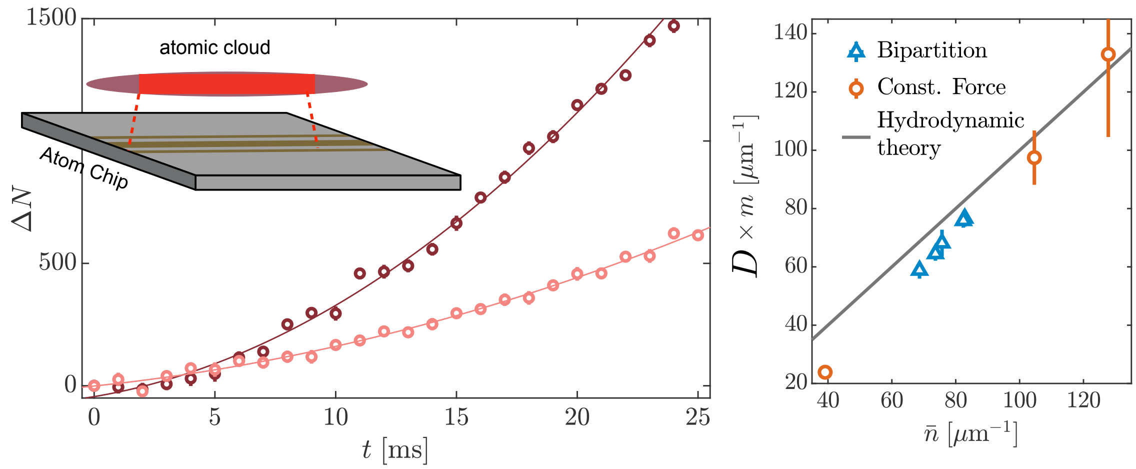
A non-zero Drude weight D then indicates the existence of a dissipationless zero-frequency mode. At finite temperature, in a generic interacting system (with Umklapp scattering to relax the momentum), we expect any delta function contribution to be broadened, so $D = 0$ and transport is fully dissipative.

The paper from the Vienna group, Schüttelkopf *et al.*, demonstrates an exceptional case of nonzero Drude weight in an interacting gas of bosonic atoms confined to one dimension (1D). Their short-range repulsion is described by the Lieb-Liniger Hamiltonian,

$$H = \frac{1}{2m}\sum_i p_i^2 + g\sum_{i<j}\delta(x_i - x_j), \tag{4}$$

which produces an integrable (as opposed to chaotic) dynamics by virtue of an infinite number of conserved quantities.*

*The integrability is elementary in the hard-core limit $g \to \infty$, when the interacting bosons may be mapped onto free spinless fermions, with the Pauli exclusion principle taking care of the hard-core repulsion. Remarkably enough, the Hamiltonian (4) remains integrable for any interaction strength, see the review on 1D bosons by Cazalilla *et al.*



Integrability-protected dissipationless transport was explained by Zotos, Naef, and Prelovšek (1997), as a consequence of the lower bound† for the Drude weight in the presence of a set of independent conserved charges Q_n:

$$2k_{\rm B}TD = \lim_{t\rightarrow\infty}\frac{1}{tL}\int_0^t \langle I(t')I(0)\rangle\, dt' \geq \frac{1}{L}\sum_n \frac{|\langle IQ_n\rangle|^2}{\langle Q_n^2\rangle}, \tag{5}$$

in a system of size L, with $\langle\cdots\rangle$ the equilibrium expectation value at temperature T. A finite right-hand-side of the inequality remains in the thermodynamic limit $L\rightarrow\infty$ if each expectation value scales $\propto L$ and if $\langle IQ_n\rangle \neq 0$ for some n, so if the current I is correlated with some conserved charge Q_n.

Evidence for dissipationless transport in an interacting 1D Bose gas was reported by Ronzheimer *et al.* (2013), but without a direct measurement of the Drude weight, as now reported by the Vienna group.

In their experiment an atomic cloud of Rb atoms is confined to a cigar-shaped magnetic trap and cooled to temperatures near 10 nK. Only the lowest transverse sub-band is occupied, the system is effectively 1D. Optical dipole potentials are superimposed onto the magnetic trap along the longitudinal axis, over a length $L = 100\,\mu$m. The system is quenched at time $t = 0$ by application of a chemical potential difference $\Delta\mu$ (constant force $\Delta\mu/L$). The subsequent evolution of the system is probed by measuring the time dependent atomic density.

The figure (left panel) shows the difference $\Delta N(t) = N_{\rm L}(t) - N_{\rm R}(t)$ of the number of atoms between the left and right halves of the system, for two different mean densities $\bar{n}$ and potential differences. The density measurements are destructive, for each data point the quench is repeated. Quadratic fits of the imbalances are plotted as solid lines. The t^2 growth of $\Delta N(t)$ implies a current $I(t) = \frac{1}{2}\Delta N'(t)$ that increases linearly with time, demonstrating ballistic (dissipationless) transport.

†The inequality (5) for the current correlator is known as the Mazur bound. Let me add a personal observation. Peter Mazur was my Ph.D. advisor. I never heard him mention the 1969 paper in which he derived this inequality, and it had no impact in the literature until 1997. I find this one of the wonderful things about our profession: A result that is true and nontrivial may lie dormant for decades, but at some moment it will awaken.

The Drude weight, obtained from

$$D = \lim_{\Delta\mu\to 0} \lim_{t\to\infty} \frac{I(t)}{t\Delta\mu} \approx \frac{\Delta N''(t)}{2\Delta\mu}, \tag{6}$$

is also plotted in the figure (right panel), both for the constant-force quench and for an alternative "bipartition" quench (consisting of a potential step halfway the trap that is suddenly flattened at $t = 0$). The density dependence follows closely the linear $\bar{n}$-dependence of the ideal-gas Drude model. A generalized hydrodynamics for integrable systems provides the theoretical framework for these experiments. The authors show that their measurements of the Drude weight are in good accord with that theory.

More complex behaviors are predicted in other integrable systems. As discussed by Karrasch, Prosen, and Heidrich-Meisner, in a 1D Fermi-Hubbard gas the spin transport is ballistic ($D_{\rm spin} \neq 0$), but the charge transport at half-filling remains diffusive ($D_{\rm charge} = 0$ because particle-hole symmetry enforces $\langle I_{\rm charge} Q_n \rangle = 0$ for all conserved charges). A Drude weight measurement would then demonstrate the unusual coexistence of ballistic and diffusive transport.