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The alphabet of superconducting circuits

Enumeration of all superconducting circuits up to 5 nodes
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Recommended with a Commentary by Anton Akhmerov, Kavli Institute of Nanoscience, Delft University of Technology

The history of superconducting qubits is a balance between slow and quantitative improvements of how one operates and makes a device and disruptive changes when a different type of qubit is discovered. The field went from an idea of a Cooper pair box, which fared terribly, to the first really functioning flux qubits. It gained speed when the more noise-resilient transmon qubit was discovered, and now many research groups switch to even more sophisticated fluxonium devices. The ideas behind each development were profound: changing the quantum degree of freedom from the charge of a single Cooper pair to current, then to the magnitude of its fluctuations, and finally to the magnetic flux. On the other hand, each of those qubits is a small electric circuit combining only three elements: inductances, capacitances, and Josephson junctions. While giving all the deserved credit to multiple physics ideas and engineering developments that made these devices possible, I want to emphasize the impact of something as simple as considering a different circuit.

The paper by Weissler *et al.* presents an approach to the field of superconducting devices that I have not considered. It presents an algorithm for enumerating all possible sufficiently small superconducting circuits. Do not be misled by the “up to 5 nodes” in the paper title: each two nodes may be connected by any combination of a Josephson junction, an inductance, and a capacitance in parallel, which gives 7 possible types of links between the nodes, or 8 if we count the absence of a connection. For a 5-node circuit whose full graph has 10 edges, this gives $\sim 8^{10}$ possible circuits. This is where the real work begins. The authors remove duplicate circuits that are equivalent from the graph theory perspective, and those whose linear parts can be transformed into one another. They then generate symbolic Hamiltonians for each circuit, and further group the circuits into equivalence classes, each class having the same Hamiltonian. This groups all circuits into a much more manageable number of 10000 inequivalent groups, although this number may still be reducible further by considering further transformations of the involved degrees of freedom.

Having the complete set of circuit Hamiltonians, the authors combine it with a search for optimal qubit designs. Using numerical optimization to get a better qubit and gate performance is a relatively well-known approach in the field, and the manuscript presents an overview of earlier works. Because evaluating qubit performance requires a combination of numerical optimization with relatively detailed simulations of both the dephasing rate, and the minimum gate duration, finding an optimal qubit is only feasible for smaller 3-node circuits. The search finds a couple of new designs that outperform the optimized fluxonium qubit by around 50%, although the authors only consider a crude performance metric. While the improvement is not dramatic, in my opinion it serves as a clear demonstration of the feasibility and promise of the enumeration approach.

Looking further, I am excited about the applications of the approach that go beyond making of the best qubit. A combination of full enumeration with graph- and group-theoretical methods was successfully used in condensed matter to classify all possible topological materials, see for example the work by Bradlyn *et al.* [1]. This suggests that instead of optimizing the lowest two levels we could use the same idea to find superconducting circuits that realize different topological Hamiltonians [2]. We already know that depending on the control parameters, the Hamiltonians of superconducting circuits belong to different symmetry classes and break the equivalent of time reversal and inversion symmetries [3]. Combining these ideas together, I envision the possibility of combining the topological classification of Hamiltonians with the knowledge of which circuits realize them as an emerging technique in the search for new devices and phenomena. To return to the title of this commentary, while only the alphabet is insufficient to make a story, knowing it helps a lot.

References

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