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The new quantum anomalous Hall effects require new concepts

1. **Extended Quantum Anomalous Hall States in Graphene/hBN Moiré Superlattices**
   Authors: Z. Lu, T. Han, Y. Yao, J. Yang, J. Seo, L. Shi, S. Ye, K. Watanabe, T. Taniguchi, and L. Ju
   arXiv:2408.10203
2. **Interplay of electronic crystals with integer and fractional Chern insulators in moiré pentalayer graphene**
   Authors: D. Waters, A. Okounkova, R. Su, B. Zhou, J. Yao, K. Watanabe, T. Taniguchi, X. Xu, Y.-H. Zhang, J. Folk, and M. Yankowitz
   arXiv:2408.10133
3. **Displacement field-controlled fractional Chern insulators and charge density waves in a graphene/hBN moiré superlattice**
   Authors: S. H. Aronson, T. Han, Z. Lu, Y. Yao, K. Watanabe, T. Taniguchi, L. Ju, and R. C. Ashoori
   arXiv:2408.11220
4. **Electric field control of superconductivity and quantized anomalous Hall effects in rhombohedral tetralayer graphene**
   Authors: Y. Choi, Y. Choi, M. Valentini, C. L. Patterson, L. F. W. Holleis, O. I. Sheekey, H. Stoyanov, X. Cheng, T. Taniguchi, K. Watanabe, and A. F. Young
   arXiv:2408.12584

Recommended with a Commentary by Trithep Devakul,
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Rhombohedral multilayer graphene (RMG) aligned with hexagonal boron nitride (hBN) has drawn great attention since last year with the discovery of the fractional quantum anomalous Hall (FQAH) effect [1], resulting in two recent Journal Club commentaries [2, 3]. This commentary is devoted to highlighting even more surprising updates to this story.

Background. RMG is an untwisted metastable configuration of N-layer graphene in a staircase configuration (also see commentary by Vishwanath [2]). The highlighted papers report new data on RMG, where the top side is aligned with a small twist angle to hexagonal boron nitride (hBN), generating a moiré superlattice.

A useful caricature of the single-particle physics of the RMG/hBN system is illustrated in Fig 1. A key parameter is Δ, an electrostatic potential difference between the top and bottom layers that can be controlled experimentally via a vertical displacement field D. Since the moiré is only present at the top layer, there are two distinct physical regimes based on the sign of Δ:

$\Delta < 0$: The moiré-distant regime. The conduction band edge is localized mostly on the layer furthest from the moiré. It is weakly affected by the moiré potential, largely resembling the pristine RMG bands.

$\Delta > 0$: The moiré-proximal regime. The conduction band edge is localized mostly on the layer closest to the moiré. It is strongly affected by the moiré potential, opening up minigaps and leading to minibands.

Surprisingly, the integer and fractional QAH effects in rhombohedral 5-layer graphene (R5G) on hBN were observed only for the moiré-distant regime [1]. This contrasts with other moiré materials, where the moiré significantly influences the formation of flat Chern bands. Here, interactions itself form the flat Chern band in a fundamental way (going beyond simply spin or valley polarization) [4–15].

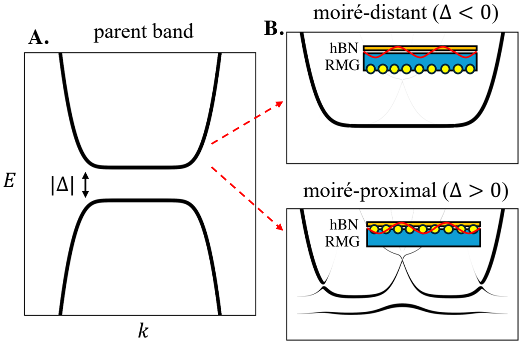

Figure 1: Sketches of electronic structure. (A) Band structure of pristine RMG near the K point. (B) The RMG/hBN spectral functions in the moiré-distant and moiré-proximal regimes. Insets show illustrations of the layer-polarized electrons.

As summarized by Parameswaran [3], the $C = 1$ state is connected to an "anomalous Hall crystal" (AHC), which breaks continuous translation symmetry to form a $C \neq 0$ state at zero magnetic field.

This makes the fractional states even more intriguing. Fits of the temperature dependence of the resistivity data reveal FQAH gaps that appear to be fraction-independent [16] (the fitted gaps are all $\sim 5\,\text{meV}$ regardless of the fraction). This goes against expectations from composite fermion theory of the fractional quantum Hall effect that gaps at $\nu = \frac{p}{2p+1}, \frac{p+1}{2p+1}$ should scale as $\frac{1}{2p+1}$. This puzzle hints at the presence of new physics.

The new experiments. The highlighted papers describe the next chapters in this story. They examine RMG/hBN in new regions of phase space, revealing unexpected new physics along with more questions:

- Lu et al. re-examine R5G/hBN (11.5 nm moiré period) in the moiré-distant regime in the same device where FQAH was previously observed [1], but at lower temperatures ($\sim 40\,\text{mK}$, compared to 400 mK previously) and find something unexpected. At low temperatures, the $C = 1$ QAH at filling $\nu = 1$ expands into an "extended QAH" (EQAH) that persists for an extended density range spanning $\nu \approx 0.5 - 1.1$. This EQAH replaces the FQAH in parts of the $\nu - D$ phase diagram. The EQAH also exhibits unusual non-linear transport — it disappears above a small threshold current.

- Waters et al. also examine R5G/hBN (10.8 nm moir'e period) in the moir'e-distant regime. They report no FQAH but find $C = 1$ QAH at $\nu = \frac{2}{3}, 1$. In a magnetic field, several $C = 1$ Chern insulators (CIs) appear at fractional densities indicating broken discrete lattice translation symmetry, as well as signs of a finite-field fractional Chern insulator (FCI) at $\nu = \frac{2}{3}$. They also report extremely interesting and mysterious features on the hole side near $\nu \approx -2.45$ with large anomalous Hall and with many features that look like quantum oscillations (but at zero magnetic field).

- Aronson et al. examine R5G/hBN (12.4 nm moiré period) in the moiré-proximal regime, where no QAH was seen previously, but in a finite magnetic field. They find that a small field can stabilize a $C = -1$ CI at $\nu = 1$ as well as FCIs at $\nu = \frac{1}{3}, \frac{2}{3}$. By tuning D and B field, they find several trivial and topological $C = -1$ states that extrapolate to fractional densities, as well as displacement-field tuned transitions between FCI and trivial insulators.
- Choi et al. examine R4G/hBN (14.6 nm moiré period). They find many interesting phases for both electron and hole doping in the moiré-distant regime: a superconducting pocket with $T_c \approx 60\,\mathrm{mK}$ around $\nu \approx -3.5$, QAH with $C = -4$ (at $\nu = -1$) and $C = 1$ (at $\nu = 1$), as well as an FCI at $\nu = \frac{2}{3}$ that persists to zero B. The $C = -4$ QAH can be electrically switched by D. They also study a $\mathrm{WS_2}$/R4G/hBN device in which they report an additional superconducting pocket attributed to spin-orbit proximity effect. The existence of superconductivity and QAH in the same device could prove important for engineering interfaces between such states in the future.

These experiments make it clear that RMG/hBN is an exceptionally rich platform where exotic phases seem to be commonplace. Both Lu et al. and Choi et al. report FQAH in R4G/hBN sample which, combined with earlier reports of FQAH in R6G/hBN [17], show that five layers is not strictly required. Each of these experiments alone are interesting enough to warrant a full commentary, but I will focus my discussion on just a few of the topics I find most pressing for future attention.

The extended QAH. The appearance of the EQAH phase is a surprise, and its nature is still unknown. It is tempting to speculate that the EQAH is evidence of an AHC: indeed, the AHC can persist over a broad density range by continuously adjusting its lattice constant, and can also explain the observed non-linear current-voltage relation due to de-pinning effects. If true, then EQAH should be realizable in RMG with no moiré at all — while a most exciting possibility, this has yet to be reported. As Lu et al. point out, another possible candidate is a Wigner crystal of holes on top of the $\nu = 1$ QAH background. Another possibility is a CI locked on to a nearby commensurate filling (like those observed by Waters et al.) but with interspersed electron or hole defects. Other possibilities exist. Ultimately, to distinguish them, more direct experimental probes that can directly probe the underlying charge order are needed. Regardless of its true identity, the EQAH appears to be an exotic new phase of matter with a quantized anomalous Hall conductance over a very broad, continuously tunable, range of densities.

The role of moiré: strong, weak, absent. There remains much to be understood about the role of the moiré potential in RMG/hBN [3]. As a thought experiment, imagine a unified phase diagram in which the strength of the moiré potential is tuned from strong (resembling moiré-proximal physics) to weak (moiré-distant) to absent (pristine RMG, unaligned to hBN). How would the physics that have been observed in these limits connect to each other?

Topological phases have mostly been observed at weak moiré. However, Aronson et al.'s observation shows that topology is also present at strong moiré, albeit different. These states appear only at finite B field, likely due to detailed energetic competition as seen in twisted bilayer graphene [18, 19]. They also following the Streda relation with *negative* $\sigma_{xy} = (-\frac{1}{3}, -\frac{2}{3}, -1)\frac{e^2}{h}$ for $B > 0$. Are these new states distinct from their moiré-distant counterparts, or are they manifestations of the same underlying physics but just with energetic differences? An appealing picture is that the Chern band in the weak moiré regime is generated by interactions, while in the strong moiré regime it is generated at the single-particle level by the moiré potential (see Fig 1). These two scenarios might be distinguished based on the sign of Chern number within a valley [13]. The opposite sign of Streda relation could reflect either (i) the Chern number or (ii) the orbital magnetic moment, within a valley, flipping sign as the potential is turned from weak to strong. The former would imply that the two limits are distinct topological phases since the Chern number is opposite for the same valley polarization.

For hole doping, another plethora of phenomena emerges at weak moiré: Waters et al. report an interesting region with anomalous Hall effect and mysterious oscillations, while Choi et al. find a $C = -4$ QAH state and superconducting pockets [20]. Could Waters et al.'s region be related to a previously observed orbital multiferroic region in moiré-less R5G [21]? Is the $C = -4$ state interaction-driven, moiré-driven, or something in between? Are the superconducting pockets related to those observed in some moiré-less RMG [22, 23] (also see commentary by Guinea [24])?

Some of these questions could potentially be probed experimentally by utilizing a remote superlattice (e.g. Refs [25, 26]) instead of the hBN moiré. This could offer a more continuous handle on the moiré potential strength.

The need for new theoretical developments. Understanding the competition of integer and fractional states, and the effect of temperature or current, calls for new theoretical developments. The lack of an isolated miniband in this system poses a serious challenge for existing numerical methods. Hartree-Fock theory cannot describe fractional states. Numerical diagonalization (or DMRG [27]) requires projection to a variational subspace of a small number of minibands, which introduces difficulties such as choosing the "best" minibands [10] and incorporating important band mixing effects [9]. Additionally, Berry curvature creates a sign problem for many Monte Carlo methods. Furthermore, there are still debates about the model itself, such as parameter choice or the treatment of interactions [8, 10]. Future progress may lie in developing new theoretical techniques or exploring simplified limits where analytic progress is possible and concentrating on universal aspects of the physics.

There is much to be understood about the general phenomenology of the topological states in this system. In an ideal quantum Hall effect, a quantized R_{xy} is accompanied by a vanishing R_{xx}. In R5G/hBN, the FQAH [1] exhibits quantized R_{xy}, but R_{xx}, despite showing a dip, remains relatively large ($R_{xx} \gtrsim 0.3\frac{h}{e^2}$). This residual R_{xx} and unusual fitted gaps [16] could be related to the presence of the closely competing EQAH state — this competition, and the role of temperature or current, therefore needs to be understood. New theories [28, 29] may contain some answers. To this end, it would be informative to see if there are qualitative differences in the transport properties of the moiré-distant and proximal FCIs, which likely have different origins.

These exciting new discoveries raise many intriguing open questions and put a spotlight on new physics that calls for new concepts to be understood. With the rapid pace at which this field is advancing, I expect that the next big breakthrough will come sooner than later.

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