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Balloon Avalanche Hysteria

Bifurcations of Inflating Balloons and Interacting Hysterons
Authors: Gentian Muhaxheri and Christian D. Santangelo
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Recommended with a Commentary by Yair Shokef, Tel Aviv University

Hysteresis is well understood for instance in ferromagnets, where the magnetization does not depend only on the magnetic field, but also on its history (the word *history* comes from a completely different source than the word *hysteresis*, not to mention *hysteria*, which is in the title just to make you read this commentary). We all use hysteresis in magnetic memory storage devices, which require energy only to change the states of their bits, but not to retain their state. Current research on the non-equilibrium responses of complex and disordered soft-matter systems [1] such as amorphous solids [2], crumpled sheets [3], origami [4], and mechanical metamaterials [5–7] identifies the origin of memory effects to be the presence of multiple and sometimes elusive, hysteretic elements, referred to as *hysterons* within the material. These can be magnetic spins, but also buckled elastic elements or creases in sheets.

Each hysteron's response is described by its switching fields, namely the values of the external field at which this hysteron switches its discrete state. If the hysterons within the system do not interact with each other, once the external field crosses the switching field of one (or more) hysteron, it switches, and this does not affect the remaining hysterons. However, if there are interactions between hysterson, once one of them switches, this can modify the switching fields of other hysterons, such that even without further changes in the field, some of them are now beyond their switching field, thus they are unstable and will immediately switch. These *avalanches*, in which switching one hysteron causes other hysterons to follow may explain emergent cooperative effects in the system.

One way to describe the history dependence of such systems is by analyzing the graphs of possible transitions between all the collective states that the system can be in [8, 9]. This enables to understand how possible graph topologies give different dynamical responses. But we still have very limited understanding of how the mechanics of a system determines the graph topology, switching fields, and interactions between hysterons.

In the recommended paper, Muhaxheri and Santangelo suggest a concrete physical system of connected inflating rubber balloons, for which they can relate the discrete framework of hysterons with the topology of continuous solutions in the system's configuration space. Due to rubber's nonlinear elasticity, the pressure in each balloon is non-monotonic as function of its volume [10]. If the pressure in all balloons is externally controlled, say by an air pump, as

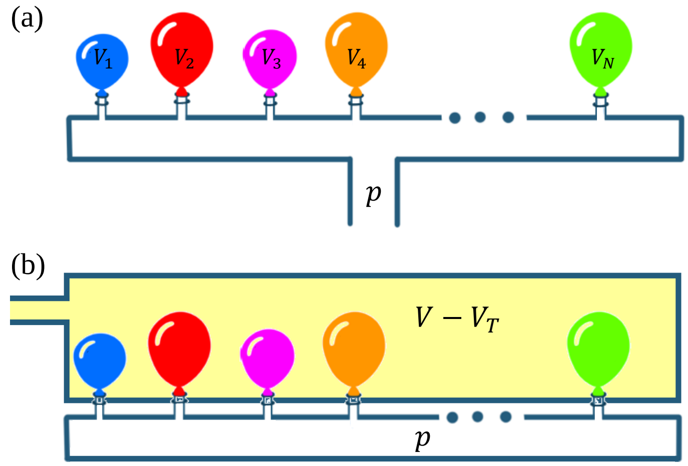

Figure 1: a) N different rubber balloons are inflated by controlling their pressure p, such that they attain volumes $\{V_i\}$. b) By placing the balloons in a tank of volume V, which is filled with a controlled amount $V - V_T$ of water (marked in yellow), their total volume $\sum V_i = V_T$ is set, while the same (uncontrolled) pressure p is maintained in all the balloons. Adapted with permission from the recommended paper.

shown in Fig. 1a, we should consider the volume of each balloon as function of the pressure. This hysteretic curve, shown in Fig. 2a motivates to identify each balloon i as a hysteron that can be in one of two states, $s_i = 0, 1$. However, in this way, the balloons do not interact, and each one independently switches its state as the pressure is increased or decreased.

Interestingly, if the total volume of all balloons is controlled, they behave as interacting hysterons. This may be obtained, for example as shown in Fig. 1b, by sealing the air pipe and placing the balloons in a water tank with a controlled amount of water, which is incompressible. Now, an individual balloon may also be in the intermediate state $s_i = \frac{1}{2}$ shown in Fig. 2b. Since the volumes of the individual balloons should give the same pressure in all of them, the system's total volumes where each balloon switches states depend on the states of all other balloons. As the total volume is ramped up or down, the system follows a curve representing the physical solution in configuration space. The geometry and topology of these curves determine the transition graph between the system's discrete states. For the simple case of two balloons, the system's configuration space is merely the $V_1 - V_2$ plane shown in Fig. 2c, and discontinuous jumps between branches in configuration space correspond to the avalanches in the transition graph seen in Fig. 2d.

The two approaches that Muhaxheri and Santangelo employ in the recommended paper for studying this concrete physical system provide complementary insights into the geometrical and topological origins of memory in disordered systems. Coming from a condensed-matter or statistical-physics perspective, it would be interesting to see how these methods may be used for analyzing larger systems with more hysterons. Constructing the transition graph directly from the switching fields provides a well-defined scheme that could be automated and in principle used for arbitrary N. The configuration-space approach allows to visualize the dynamics and enables to identify and characterize bifurcations, where changes

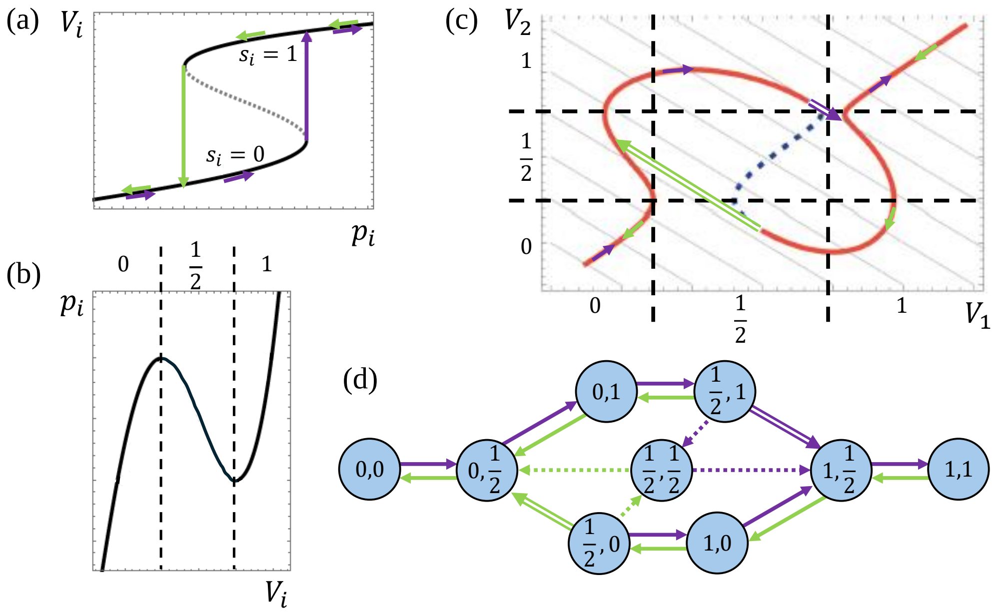

Figure 2: a) Since pressure *vs.* volume in a rubber balloon is non-monotonic, under pressure control, volume is hysteretic; for given pressure, depending on history, it may be in two possible states $s_i = 0, 1$. b) Under volume control, the balloon can also be in the intermediate state $s_i = \frac{1}{2}$. c) The configuration space of two balloons under volume control, $V_1 + V_2 = V_T$ (slanted gray lines). The requirement $p_1(V_1) = p_2(V_T - V_1)$ leads to a solution with stable (red line) and unstable (dashed blue line) segments. Upon increasing (decreasing) V_T, the system jumps between the two stable branches, as shown by the double purple (green) line. d) Corresponding transition graph between discrete states. Double arrows indicate avalanches, where the system goes (dashed lines) to an unstable state and immediately continues to a stable state. Adapted with permission from the recommended paper.

in the transition-graph topology occur. However, the latter approach may require more manual work, and without developing further methods it may become computationally and theoretically too complicated to handle as system size increases.

It would be interesting to understand if for larger systems, additional phenomena appear, and to what extent an extended system may be understood based on motifs from smaller subsystems within it. Similarly to recent works on designing bifurcations in configuration space of mechanical systems [11], here transitions and avalanches in the system may be designed via bifurcations of solution curves in configuration space, and it would be interesting to employ this configuration-space approach to other physical systems such as buckled beams, origami, or even amorphous solids and glassy systems. Richer responses appear when more than one external field or control variable can drive the system, in different sequences. This too could be included in the analysis of the dynamics in configuration space. Finally, correlations between the behavior of interacting hysterons appear due to similarities or connections between the $p_i(V_i)$ functions of the different balloons, like where their minima occur, and not due to physical interaction. This could lead to new perspectives on interacting hysterons.

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