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Non-universal edge state physics in the quantum Hall effect

1. **Direct visualization of electronic transport in a quantum anomalous Hall insulator**
   Authors: Ferguson, G. M.; Xiao, Run ; Richardella, Anthony R. ; Low, David ; Samarth, Nitin; Nowack, Katja C.
   Nat. Mat. **22** 1100-1105 (2023)
2. **Signature of anyonic statistics in the integer quantum Hall regime**
   Authors: P. Glidic, I. Petkovic, C. Piquard, A. Aassime, A. Cavanna, Y. Jin, U. Gennser, C. Mora, D. Kovrizhin, A. Anthore, F. Pierre
   arxiv:2401.06069

Recommended with a Commentary by Roderich Moessner,
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General background— The past decades have seen an almost inexorable move towards universal physics, at the expense of interest in details. The great success of the renormalisation group and the principles of hydrodynamics have underpinned a systematic and powerful understanding of collective phenomena in the limit of long time- and lengthscales. For these insights it is often neither necessary, nor even particularly useful, to dwell too much on what's going on on the lattice scale.

This development has been reinforced further by the advent of the notion of topology. Certain phenomena, most prominently Klitzing's quantised Hall conductance e^2/h, are not only absolutely stable in the sense of being impervious to just about any reasonable local perturbation. Rather, they are so robust that a quantitative calculation on a *wrong* microscopic model will give a 'correct' result for the topologically protected quantity under consideration. By 'wrong' here I mean studying a model whose microscopic properties are in fact at qualitative variance with what actually happens in experiment.

A counterveiling trend to these developments has been set by the availability of increasingly powerful experimental probes, in particular those emanating from nanophysics. Concretely, the availability of micro-SQUIDS or NV-centres on a tip allows the measurement of magnetic fields, and hence the inference of current density distributions, on (even sub)micron lengthscales, while nanopatterning allows for increasing microscopic control on the nanoscale. It is thus natural to ask questions about the physics these probes unveil, much of which takes place away from the universal/topological physics mentioned above, in the sense that it contains genuinely additional information beyond long time and lengthscales.

Before outlining the experimental results, it is necessary to emphasize that this does not mean that nobody has been interested in such questions. Indeed, there exist pioneering antecedants dating back a long time. One is a sustained research effort by Kltizing's group on the current distribution in the quantum Hall effect [1]; the other a theoretical proposal for non-trival internal structure of integer quantum Hall effect edge states [2]; and much of the discussion below builds on a long research effort in building quantum Hall interferometers.

What the results discussed here have in common is that they are interested in what happens on the edge of a system with a topologically quantised Hall conductance, i.e. precisely the kind of non-universal question mentioned above.

Experiment by Ferguson et al.: where does the current flow?— The picture of current carried by narrow edge states has considerable popularity, following seminal pioneering work by Halperin and Wen, and above all the success in accounting for experiments such as the one discussed in the next paragraphs. Nonetheless, it is always worth bearing in mind that the simple picture of the current being carried by a narrow edge channel may, quite simply, be incorrect for a given system. A case in point is provided by experiments on a Chern insulator, Cr-doped $(\mathrm{Bi,Sb})_2\mathrm{Te}_3$. This exhibits a quantum anomalous Hall effect, where the nonequilibrium current (Fig. 1) is carried by an 'edge state' whose breadth can be much larger than a microscopic lengthscale, is variable, and whose location can be tuned to lie deep in the sample. When the two counterpropagating 'edge states' eventually meet in the bulk, the quantisation breaks down as expected.

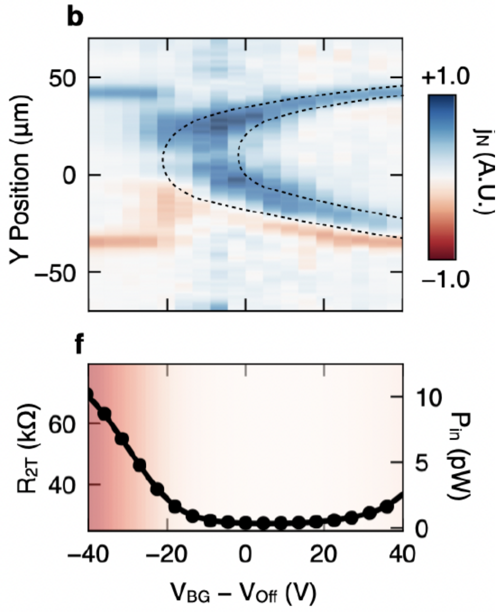

Figure 1: Transport current distribution through a cut across the sample (vertical axis) in the QAHE as a function of backgate voltage (horizontal axis), i.e. electron density. For values where the resistance takes on a quantised value (bottom panel), the non-equilibrium current flow is distributed broadly, and tunably, across the sample.

Experiment by Glidic et al.: fractional charge on integer quantum Hall edges— This experiment considers correlations in the current noise at two terminals of a nanopatterned

GaAs sample, allowing controlled access to a set of edge states, Fig. 2. It interprets anticorrelations as signatures of anyonic statistics of the objects carrying the current in the edge states, i.e. as a signature of fractionalisation even in an integer quantum Hall system.

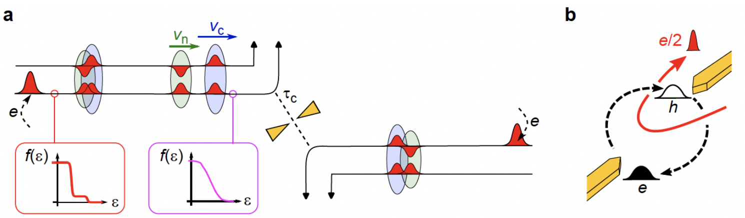

Figure 2: Set-up for noise measurement on edge magnetoplasmons. An electron tunnelling into a single edge state decomposes into two magnetoplasmons with different propagation speeds and non-integer charge in each edge state of the channel. Tunnelling across a quantum point contact induces correlations in the noise measured at the drains of the edge states.

Central ingredient is the existence a channel involving two edge states (between the vacuum and the two spin copies of the lowest Landau level), which are coupled electrostatically. The following is a brief verbal description of the bosonized treatment of the chiral Luttinger theory for these two edge states, see e.g. Ref. [2]. Tunnelling takes place into only one of the two, while the eigenfunctions involve both of them, so that two excitations of the channel are generated which have different charge distributions on the two edge states. For strongly coupled edge states, the excitations carry charges $(e/2, \pm e/2)$ on the two edge states, respectively, i.e. one is a neutral, the other a charged, edge magnetoplasmon. These co-propagate, but with differing velocities, so that they separately arrive at a downstream quantum point contact with (tunable) transmission τ_c.

The tunnelling between the edge states belonging to the two channels meeting at the quantum point contact then induces correlations in the noise measured at the drains of the channels. This noise can be calculated in detail using bosonisation/refermionisation techniques, and a result is given by the purple line in Fig. 3. The theoretical treatment is fairly complete, and agrees reasonably well with experiment.

Why I find this work interesting— These are beautiful experimental results, and it is amazing every time how detailed a picture can be obtained from such well-designed and highly tunable set-ups. These experiments were done very carefully, identifying relevant parameters (timescales, temperatures etc) and are open to quantitative modelling. They provide results which are qualitatively striking.

The theory calculation of the noise does not lend itself to an easy interpretation in terms of physical pictures, and that manuscript offers an impressionistic picture based on what is known as 'braiding in time', a notion which is well-described (including a little movie in the SM) in Ref. [3]. The idea is that a particle-hole excitation of the quantum Hall liquid in the bulk around the quantum point contact may spontaneously arise, Fig. 2b, and be separated, to be swept away along the the edges.

The edge magnetoplasmons arriving at the drains may then effectively follow, or precede, such an excitation. When these particle-hole pair creation events are rare, a perturbative treatment is possible in which the corresponding two amplitudes add up to give the observed signal. The interference between the two can be associated with a phase. It is this phase

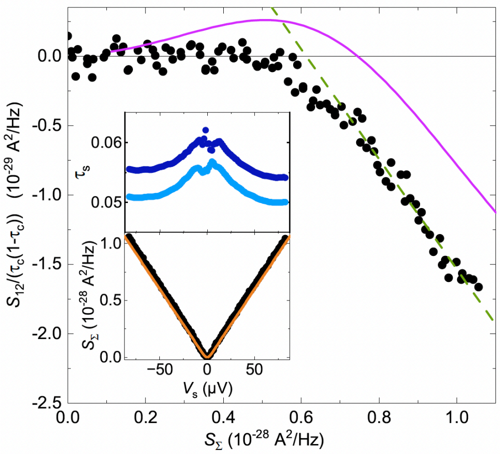

Figure 3: Main panel: noise correlations between the drains of the edge states (solid arrows pointing up/down in Fig. 2a) for equal current injections at the sources (dashed arrows in Fig. 2a), as function of noise at source S_Σ. Purple line is a non-equilibrium calculation for chiral Luttinger liquid.

which is interpreted as a anyonic exchange phase in this work–it would vanish for fermions or bosons, but is non-zero for the fractional charge of the magnetoplasmons.

Note that it is not non-fractionalised quasiparticles in the bulk that are ascribed the anyonic phase, but the fractionally charged magnetoplasmons in the edge states. This is how the notion of fractionalisation enters the IQHE regime.

The question one is left with is: how compelling is such an interpretation in terms of anyon braiding? The chiral Luttinger liquids are gapless, so that there is a priori no obstacle to a continuously tunable (i.e., in general even irrational) charge. This is made explicit in the original theory, Ref. [2], where even in the absence of tunnelling through a quantum point contact, a continuously tunable parameter encoding details of the spatially variable interaction between the channels appears in the expression for the fractional charge. The present experiment could presumably in principle also vary the strength of the coupling between the edge channels, and hence continuously tune an analogous parameter.

The term braiding in this context perhaps conjures up a mental picture borrowed from braiding quantum Hall quasiparticles, with the anyonic phase arising via the cooperative physics of the incompressible quantum Hall bulk [4]. While the origin of the chiral nature of the edge states in the present work also relies on the underlying quantum Hall effect, the physical description may to some observers suggest the language of operator commutation relations rather than that of braiding: when the order of creation operators of excitations of a chiral Luttinger liquid are interchanged, they pick up a phase which depends continuously on the interaction parameter of the Luttinger liquid.

Ultimately, these are questions of taste, in that the results presented here exist only in a window of finite frequencies. A sharp definition of fractionalisation and braiding (see e.g. Chapter 1.8 of Ref. [5]) may not be what the present set-up satisfies, but that point is to a certain degree moot: non-universal physics can be defined without strict reference to long time and distance scales, and may therefore be 'connected' non-uniquely to phenomena defined in those settings. The individual transient regimes may nonetheless, like in the present case, contain remarkable and interesting physical phenomena.

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